

# OCR

Oxford Cambridge and RSA

## Friday 20 May 2016 – Morning

### AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Find  $\sum_{r=1}^n (3r+1)(r-1)$ , giving your answer in a fully factorised form. [5]
- 2 The complex number  $z$  has modulus  $2\sqrt{3}$  and argument  $-\frac{1}{3}\pi$ . Giving your answers in the form  $x+iy$ , where  $x$  and  $y$  are exact real numbers, and showing clearly how you obtain them, find
- (i)  $z$ , [2]
- (ii)  $\frac{1}{(z^* - 5i)^2}$ . [5]
- 3 The quadratic equation  $kx^2 + x + k = 0$  has roots  $\alpha$  and  $\beta$ .
- (i) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . [1]
- (ii) Find the value of  $\left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$  in terms of  $k$ . [5]
- 4 The matrices **A**, **B** and **C** are given by  $\mathbf{A} = \begin{pmatrix} a & 2 & 3 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} b & 0 & 5 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ . Find
- (i)  $5\mathbf{A} - 3\mathbf{B}$ , [2]
- (ii)  $\mathbf{BC}$ , [2]
- (iii)  $\mathbf{CA}$ . [2]
- 5 The sequence  $u_1, u_2, u_3, \dots$  is defined by  

$$u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geq 1.$$
 Prove by induction that  $u_n = 2 \times 3^n - 1$ . [4]
- 6 In an Argand diagram the points  $A$  and  $B$  represent the complex numbers  $5 + 4i$  and  $1 + 2i$  respectively.
- (i) Given that  $A$  and  $B$  are the ends of a diameter of a circle  $C$ , find the equation of  $C$  in complex number form. [4]
- The perpendicular bisector of  $AB$  is denoted by  $l$ .
- (ii) Sketch  $C$  and  $l$  on a single Argand diagram. [2]
- (iii) Find the complex numbers represented by the points of intersection of  $C$  and  $l$ . [3]
- 7 The matrix  $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$  represents a transformation  $P$ .
- (i) Describe fully the transformation  $P$ . [2]
- The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$ .
- (ii) Given that  $\mathbf{M}$  represents transformation  $Q$  followed by transformation  $P$ , find the matrix that represents the transformation  $Q$  and describe fully the transformation  $Q$ . [6]

8 (i) Show that  $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$ . [1]

(ii) Hence find  $\sum_{r=1}^n \frac{1}{(2r+1)(2r+3)}$ , giving your answer as a single fraction. [6]

(iii) Find  $\sum_{r=n}^{\infty} \frac{1}{(2r+1)(2r+3)}$ , giving your answer as a single fraction. [3]

9 (i) The matrix  $\mathbf{X}$  is given by  $\mathbf{X} = \begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$ . Show that the determinant of  $\mathbf{X}$  is  $a^2 - 8a + 15$ . [3]

(ii) Explain briefly why the equations

$$3x + 3y - 2z = 1$$

$$3y + 5z = 5$$

$$x + 2y + z = 2$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. [3]

10 (i) Use an algebraic method to find the square roots of the complex number  $9 + 40i$ . [6]

(ii) Show that  $9 + 40i$  is a root of the quadratic equation  $z^2 - 18z + 1681 = 0$ . [1]

(iii) By using the substitution  $z = \frac{1}{u^2}$ , find the roots of the equation  $1681u^4 - 18u^2 + 1 = 0$ . Give your answers in the form  $x + iy$ , where  $x$  and  $y$  are real. [4]

**END OF QUESTION PAPER**

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