OCR
Oxford Cambridge and RSA

Friday 20 May 2016 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Find $\sum_{r=1}^{n}(3r+1)(r-1)$, giving your answer in a fully factorised form. **[5]**

2 The complex number z has modulus $2\sqrt{3}$ and argument $-\frac{1}{3}\pi$. Giving your answers in the form $x+\mathrm{i}y$, where x and y are exact real numbers, and showing clearly how you obtain them, find

(i) z, **[2]**

(ii) $\dfrac{1}{(z^*-5\mathrm{i})^2}$. **[5]**

3 The quadratic equation $kx^2+x+k=0$ has roots α and β.

(i) Write down the values of $\alpha+\beta$ and $\alpha\beta$. **[1]**

(ii) Find the value of $\left(\alpha+\dfrac{1}{\alpha}\right)\left(\beta+\dfrac{1}{\beta}\right)$ in terms of k. **[5]**

4 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (a \;\; 2 \;\; 3)$, $\mathbf{B} = (b \;\; 0 \;\; 5)$ and $\mathbf{C} = \begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$. Find

(i) $5\mathbf{A} - 3\mathbf{B}$, **[2]**

(ii) **BC**, **[2]**

(iii) **CA**. **[2]**

5 The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 5 \text{ and } u_{n+1} = 3u_n + 2 \text{ for } n \geqslant 1.$$

Prove by induction that $u_n = 2 \times 3^n - 1$. **[4]**

6 In an Argand diagram the points A and B represent the complex numbers $5+4\mathrm{i}$ and $1+2\mathrm{i}$ respectively.

(i) Given that A and B are the ends of a diameter of a circle C, find the equation of C in complex number form. **[4]**

The perpendicular bisector of AB is denoted by l.

(ii) Sketch C and l on a single Argand diagram. **[2]**

(iii) Find the complex numbers represented by the points of intersection of C and l. **[3]**

7 The matrix $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ represents a transformation P.

(i) Describe fully the transformation P. **[2]**

The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} -3 & -1 \\ -1 & 0 \end{pmatrix}$.

(ii) Given that **M** represents transformation Q followed by transformation P, find the matrix that represents the transformation Q and describe fully the transformation Q. **[6]**

8 **(i)** Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. **[1]**

(ii) Hence find $\sum_{r=1}^{n} \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[6]**

(iii) Find $\sum_{r=n}^{\infty} \frac{1}{(2r+1)(2r+3)}$, giving your answer as a single fraction. **[3]**

9 **(i)** The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} a & 3 & -2 \\ 0 & a & 5 \\ 1 & 2 & 1 \end{pmatrix}$. Show that the determinant of **X** is $a^2 - 8a + 15$. **[3]**

(ii) Explain briefly why the equations

$$\begin{aligned} 3x + 3y - 2z &= 1 \\ 3y + 5z &= 5 \\ x + 2y + z &= 2 \end{aligned}$$

do not have a unique solution and determine whether these equations are consistent or inconsistent. **[3]**

10 **(i)** Use an algebraic method to find the square roots of the complex number $9 + 40\mathrm{i}$. **[6]**

(ii) Show that $9 + 40\mathrm{i}$ is a root of the quadratic equation $z^2 - 18z + 1681 = 0$. **[1]**

(iii) By using the substitution $z = \frac{1}{u^2}$, find the roots of the equation $1681u^4 - 18u^2 + 1 = 0$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real. **[4]**

END OF QUESTION PAPER